

Compounding Swap

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Compounding Swap Introduction

- ◆ A compounding swap is a swap in which interest, instead of being paid, compounds forward until the next payment date.
- ◆ Compounding swaps can be valued by assuming that the forward rates are realized.
- ◆ Normally the calculation period of a compounding swap is smaller than the payment period. For example, a swap has 6-month payment period and 1-month calculation period (or 1-month index tenor).
- ◆ An overnight index swap (OIS) is a typical compounding swap.

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Payoff

- ◆ Assuming that a compounding swap consists of two legs: a regular fixed leg and a compounding floating leg.
- ◆ The compounding leg is similar to a regular floating leg except the reset frequency is higher than the payment frequency. For example, a compounding leg has 1-month reset frequency and 6-month payment frequency.
- ◆ From the fixed rate receiver perspective, the payoff of a swap or swaplet at payment date T is given by

$$Payff_{payer} = N\tau R - NF$$

where

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Payoff (Cont.)

- ◆ N - the notional;
- ◆ τ - accrual period in years (e.g., a 3 month period $\approx 3/12 = 0.25$ years)
- ◆ R - the fixed rate in simply compounding.
- ◆ $F = \prod_{j=1}^k (1 + Q_j) - 1$ - the realized interest payment for the payment period, say, 6-month.
- ◆ $Q_j = r_j \tau_j$ - the accrued interest for the calculation period, say, 1-month.
- ◆ r_j - the interest rate
- ◆ From the fixed rate payer perspective, the payoff of a swap or swaplet at payment date T is given by

$$Payoff_{receiver} = N (F - \tau R)$$

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Valuation

- ◆ The present value of a fixed rate leg is given by

$$PV_{fixed}(t) = RN \sum_{i=1}^n \tau_i D_i$$

where t is the valuation date and $D_i = D(t, T_i)$ is the discount factor.

- ◆ The present value of a compounding leg is given by

$$PV_{compound}(t) = N \sum_{i=1}^n \left(\prod_{j=1}^k (1 + Q_j) - 1 \right) D_i$$

where

- ◆ $Q_j = (F_j + s)\tau_j$ - the accrued interest for calculation period j .
- ◆ $F_i = \left(\frac{D_{i-1}}{D_i} - 1 \right) / \tau_i$ - the simply compounded forward rate
- ◆ s - the floating spread.

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Valuation (Cont.)

- ◆ The present value of an interest rate swap can be expressed as
 - ◆ From the fixed rate payer perspective, $PV = PV_{float} - PV_{fixed}$
 - ◆ From the fixed rate receiver perspective, $PV = PV_{fixed} - PV_{float}$

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Notes

- ◆ First of all, you need to generate accurate cash flows for each leg. The cash flow generation is based on the start time, end time and payment frequency of the leg, plus calendar (holidays), business convention (e.g., modified following, following, etc.) and whether sticky month end.
- ◆ We assume that accrual periods are the same as reset periods and payment dates are the same as accrual end dates in the above formulas for brevity. But in fact, they are different due to different market conventions. For example, index periods can overlap each other but swap cash flows are not allowed to overlap.
- ◆ The accrual period is calculated according to the start date and end date of a cash flow plus day count convention

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Notes (Cont.)

- ◆ The forward rate should be computed based on the reset period (index reset date, index start date, index end date) that are determined by index definition, such as index tenor and convention. it is simply compounded.
- ◆ Sometimes there is a floating spread added on the top of the floating rate in the floating leg.
- ◆ The present value of the reset cash flow should be added into the present value of the floating leg.
- ◆ Some dealers take bid-offer spreads into account. In this case, one should use the bid curve constructed from bid quotes for forwarding and the offer curve built from offer quotes for discounting.

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Example

Leg 1 Specification		Leg 2 Specification	
Currency	USD	Currency	USD
Day Count	dcAct360	Day Count	dcAct360
Leg Type	Fixed	Leg Type	Float
Notional	5000000	Notional	5000000
Pay Receive	Receive	Pay Receive	Pay
Payment Frequency	6M	Payment Frequency	6M
Start Date	7/1/2015	Start Date	7/1/2015
End Date	3/1/2023	End Date	3/1/2023
Fixed Rate	0.0455	Spread	0
		Index Specification	
		Index Type	LIBOR
		Index Tenor	1M
		Index Day Count	dcAct360



Reference:

<https://finpricing.com/lib/EqBarrier.html>